UK Maths Trust

INTERMEDIATE MATHEMATICAL CHALLENGE
Wednesday 31 January 2024

Organised by the United Kingdom Mathematics Trust

supported by G RESEARCH Overleaf

> *England & Wales: Year 11 or below*
> *Scotland: S4 or below*
> *Northern Ireland: Year 12 or below*

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.

2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.

3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.

4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.

5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.

6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   Each incorrect answer to Questions 16-20 loses 1 mark;
   Each incorrect answer to Questions 21-25 loses 2 marks.

7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.

8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

9. To accommodate candidates sitting at other times, please do not discuss the paper on the internet until **08:00 BST on Friday 2 February**. Candidates in time zones more than 3 hours ahead of GMT must sit the paper on Thursday 1 February (as defined locally).
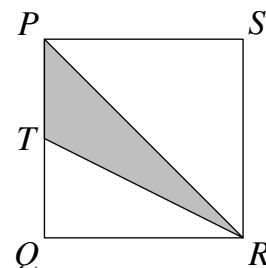
1. What is the value of $\dfrac{20 + 24}{20 - 24}$?

     A   8   B   −9   C   10   D   −11   E   12

2. What is the difference between the smallest two-digit prime and the largest two-digit prime?

     A   86   B   84   C   82   D   80   E   78

3. The diagram shows the square $PQRS$ and T, the mid-point of the side PQ. What fraction of the area of the square $PQRS$ is shaded?

     A $\dfrac{1}{4}$ B $\dfrac{1}{3}$ C $\dfrac{1}{2}$ D $\dfrac{2}{3}$ E $\dfrac{3}{4}$

4. The shorter sides of a right-angled triangle have lengths $\sqrt{5}$ and $\sqrt{12}$.
What is the length of the hypotenuse?

     A $\sqrt{7}$ B $\sqrt{13}$ C $\sqrt{15}$ D $\sqrt{17}$ E   7

5. The ages of Grannie's seven grandchildren are consecutive positive integers. The youngest three grandchildren have a mean age of 6. What is the mean age of the oldest three grandchildren?

     A   8   B   9   C   10   D   11   E   12

6. Four of these points lie on a circle. Which of the points does not lie on that circle?

   A $(5, 0)$ B $(4, 3)$ C $(2, 2)$ D $(3, 4)$ E $(0, 5)$

7. The 'Penny's Puddings' company uses one tonne of rice to make twenty-five thousand cans of rice pudding. Each tonne of rice contains approximately fifty million grains of rice.
Approximately how many grains of rice are there in a can of Penny's rice pudding?

     A   200   B   2000   C   5000   D   50 000   E   1 250 000

8. What is the value of $999 \times 999 + 999$?

     A   10 800   B   100 800   C   999 000   D   999 999   E   1 000 998

9. Three sectors of a circle are removed from a regular hexagon to form the shaded shape shown. Each sector has perimeter 18 mm.
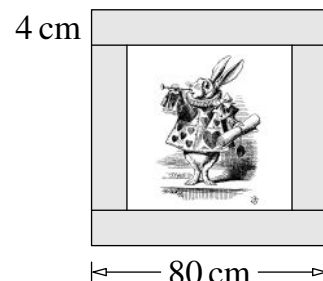What is the perimeter, in mm, of the shaded shape formed?
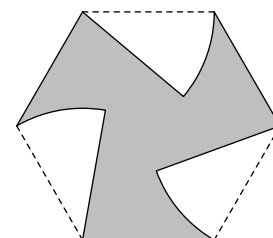
     A   48   B   50   C   52   D   54   E   56

10. Which of the following is equal to $\frac{20}{24} + \frac{20}{25}$?
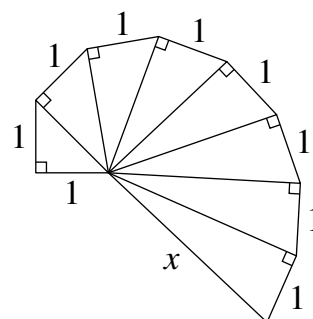
   A $\frac{40}{600}$ B $\frac{49}{30}$ C $\frac{30}{49}$ D $\frac{40}{49}$ E

$\frac{49}{40}$

11. A picture, together with its frame, makes a square of side-length 80 cm. The frame is 4 cm wide.
What percentage of the area of the square is covered by the frame?

   A 15%   B 18%   C 19%   D 20%   E 24%

12. What is the length of the line segment marked x?

   A $\sqrt{2}$ B 2   C $2\sqrt{2}$ D 3   E 4

13. When I increase a certain number by 20%, I get twice as much as when I decrease 20 less than the number by 20%. What is that number?

   A 40   B 60   C 80   D 100   E 120

14. Going clockwise around a quadrilateral, its interior angles are in the ratio $6 : 7 : 8 : 9$.
Which of the following is a true statement about the quadrilateral?

   A It has a right angle   B It is a trapezium   C It is a parallelogram
   D It is a kite   E It is cyclic

15. Carrie the cat and Barrie the bat together weigh 4000 g more than Rollie the rat.
Barrie and Rollie together weigh 2000 g less than Carrie.
Carrie and Rollie together weigh 3000 g more than Barrie.
What is the weight, in grams, of Rollie the rat?

   A 250   B 500   C 750   D 1000   E 1250

16. Factorial n, written $n!$, is defined by: $n! = 1 \times 2 \times 3 \times \cdots \times n$.
What is the remainder when $1! + 2! + 3! + 4! + 5! + 6! + 7! + 8! + 9! + 10!$ is divided by 5?

   A 0   B 1   C 2   D 3   E 4

17. What is $4^{(3^2)}$ divided by $(4^3)^2$?

   A 1   B 6   C 16   D 32   E 64

18. The point $P(-1, 4)$ is reflected in the y-axis to become Q. The point Q is reflected in the line $y = x$ to become R. The point R is reflected in the x-axis to become S.
What is the area of quadrilateral $PQRS$?

    A 4   B $4\sqrt{2} + 2$ C 6   D $4 + 2\sqrt{2}$ E 8

19. In the grid shown the three non-zero numbers in each row, each column and each diagonal *multiply* to give the same *product*.
What is the value of x?

|		
		6
2	x	3

    A 36   B 18   C 12   D 9   E 4

20. A shop sign says, "T-shirts. Three for the price of two. Equivalent to a saving of £5.50 on each T-shirt." Using this special offer, what is the cost of three T-shirts?

    A £16.50   B £22   C £31   D £33   E £49.50

21. The diagram shows a square of side $4\,\text{cm}$ with four identical semi-circles drawn with their centres at the mid-points of the sides. The four semi-circles each touch two other semi-circles, as shown.
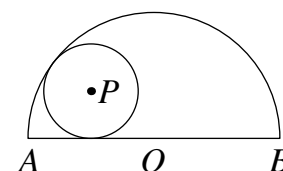What is the shaded area, in cm^2?
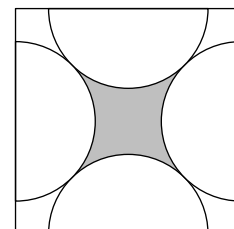
    A $8 - \pi$ B π C $\pi - 2$ D $\pi - \sqrt{2}$ E $8 - 2\pi$

22. When a cube is cut into two pieces with a single plane cut, two polyhedra are obtained.
Which of these polyhedra *cannot* be obtained in this way?

    A A polyhedron with 4 faces   B A polyhedron with 5 faces   C A polyhedron with 6 faces
    D A polyhedron with 7 faces   E A polyhedron with 8 faces

23. A circle is inscribed in a semicircle with centre O and diameter AB. The centre of the circle is the point P, where $PA = PO$.
What is the ratio of the radius of the circle to the radius of the semicircle?

    A $4 : 9$ B $3 : 8$ C $3 : 7$ D $2 : 5$ E $1 : 2$

24. The diagram shows a regular hexagon $RSTUVW$.
The area of the shaded pentagon $RSTPQ$ is one quarter of the area of the hexagon. Jay and Kay walk around the hexagon from P to Q, Jay going clockwise and Kay anticlockwise.
What is the ratio of the distance Jay walks to the distance Kay walks?
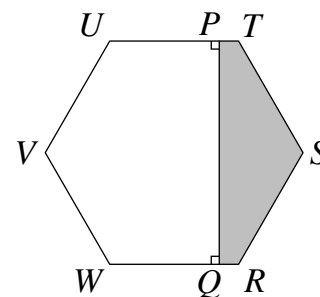
    A $1 : 2$ B $2 : 3$ C $3 : 5$ D $4 : 7$ E $5 : 8$

25. A gold coin is worth $x\%$ more than a silver coin. The silver coin is worth $y\%$ less than the gold coin. Both x and y are positive integers. How many possible values for x are there?

    A 0   B 3   C 6   D 9   E 12